Letters

Comments on "Finite-Element Analysis of Waveguide Modes: A Novel Approach That Eliminates Spurious Modes"

Michał Mrozowski

One of the drawbacks of the finite-element analysis of waveguiding structures is that it often yields nonphysical solutions, which are called spurious modes. In the above paper[1] a novel formulation of the finite element method is presented which allows one to readily identify proper modes. The approach is based on the variational expression of the propagation constant involving transverse electric and magnetic field components. The following generalized eigenvalue problem is obtained from the stationary condition:

$$\underline{\underline{Q}}\,\underline{\Phi} = \frac{1}{\beta}\underline{\underline{P}}\,\underline{\Phi}. \qquad (1)$$

The eigenvalues $1/\beta$ of the above problem are the reciprocals of the propagation constants of the modes supported by the structure under investigation. In general, the eigenvalues of problem (1) can be complex numbers. To identify guided modes only real eigenvalues are selected. All eigenvalues equal to zero and those having nonzero imaginary parts are attributed by the authors (see subsection IV-A) to "definitely nonphysical spurious-mode solutions." This conclusion cannot fully be accepted because a nonzero imaginary part of the propagation constant does not necessarily mean that the eigensolution is nonphysical.

1) In addition to guided modes, each guide may support an infinite number of higher order cutoff modes which have purely imaginary propagation constants. At least some of the solutions referred to by the authors as nonphysical could in fact have been cutoff waves.
2) Guides containing anisotropic or inhomogeneous isotropic media may support, even in the lossless case, complex waves [1], i.e., modes with complex propagation constants. Complex waves are physically admissible solutions and their omission in the discontinuity analysis may lead to serious errors [2]. Incidently, for a rectangular guide with one-dimensional inhomogeneity discussed in subsection IV-A of the paper, no complex waves are allowed. Thus for this case the solutions with complex eigenvalues might have actually been spurious, but one has to bear in mind that eigenvalues with a small real part compared with the imaginary part (type 3 in Table I in the paper in question) could also be due to round-off errors, especially in ill-conditioned problems. Problem (1) is ill-conditioned as matrix $\underline{\underline{Q}}$ is singular.

Concluding, the formulation proposed in the paper seems to enable one to identify guided modes with real propagation constants but care has to be taken when attributing the eigenvalues having a nonzero imaginary part to spurious modes.

Manuscript received August 22, 1990.
The author is with the Telecommunication Institute, Technical University of Gdańsk, 80-952 Gdańsk, Poland.
IEEE Log Number 9041940.
[1]T. Angkaew, M. Matsuhara, and N. Kumagai, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 117–123, Feb. 1987.

Reply[2] by Tuptim Angkaew, Masanori Matsuhara, and Nobuaki Kumagai[3]

Using the variational method, we derived equation (15) in our paper on the assumptions that the permittivity and permeability tensors are Hermite tensors and that the propagation constant β is real. Therefore, any solution to (15) with a nonreal propagation constant does not conform to these assumptions, and we select only solutions with real propagation constants.

However using the Galerkin method or weak formulation, (15) may be derived without the assumptions made in our paper. In this case, certain solutions with nonreal propagation constants correspond to the evanescent modes and the modes with loss or gain. These are reported in [3].

References

[1] P. J. B. Clarricoats and K. R. Slinn, "Complex modes of propagation in dielectric-loaded circular waveguide," *Electron. Lett.*, vol. 1, pp. 145–146, July 1965.
[2] A. S. Omar and K. F. Schünemann, "Effect of complex modes at finline discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1508–1514, Dec. 1986.
[3] M. Matsuhara and T. Angkaew, "Analysis of the waveguide with loss or gain by the finite-element method," *Trans. IEICE*, vol. J71-C, pp. 1398–1403, Oct. 1988 (in Japanese).

[2]Manuscript received October 1, 1990.
[3]The authors are with the Department of Electrical Communication Engineering, Faculty of Engineering, Osaka University, 2-1 Yamada-oka, Suita, Osaka 565, Japan.
IEEE Log Number 9041941.

Comments on "An Exact Solution for the Nonuniform Transmission Line Problem"

Smain Amari

As the author of the above paper[1] points out in his introduction, this problem has been solved within certain approximations. A limitation of these methods is the fact that some parameters of the line do not vary independently. It is exactly this kind of interdependence between the impedance and admittance that limits the form of lines to which the present solution

Manuscript received August 7, 1990.
The author is with the Department of Physics, Washington University, St. Louis, MO 63130.
IEEE Log Number 9041942.
[1]C. Nwoke, *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 944–946, July 1990.

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applies. The voltages and the currents are assumed to satisfy the following equations:

$$V'(x) = -Z(x)I(x) \tag{1}$$

$$I'(x) = -Y(x)V(x). \tag{2}$$

These equations can be written in a matrix form, as in eq. (3) of the paper in question, the solution to which is claimed to be

$$W(x) = \exp\left\{\int_{x_0}^{x} A(t)\,dt\right\} W_0 \tag{3}$$

where $W(x)$ is a column vector with components $V(x)$ and $I(x)$, and $A(x)$ is a 2×2 matrix. It should be pointed out at this point that this is true only if the matrix satisfies the following condition: the matrices $A(x_1)$ and $A(x_2)$ must commute for all x_1 and x_2. To see this, we expand the exponential in its power series (since this is how a function of an operator is defined) to obtain

$$\exp\int_{x_0}^{x} A(t)\,dt = \sum_{n=0}^{\infty}\frac{1}{n!}\left\{\int_{x_0}^{x} A(t)\,dt\right\}^n. \tag{4}$$

Differentiating this series with respect to x (up to second term for clarity), we get

$$A(x) + \frac{1}{2!}\left\{A(x)\int A(t)\,dt + \int A(t)\,dt A(x)\right\} + \cdots. \tag{5}$$

It is clear that (5) reduces to (3) only if the matrix $A(x)$ satisfies the following condition:

$$A(x)A(y) - A(y)A(x) = 0 \tag{6}$$

or, equivalently, that the ratio $Z(x)/Y(x)$ be a constant independent of the position x. Furthermore, the generalization of the stated theorem as presented by the author applies only to matrices satisfying this condition. A constant matrix obviously satisfies condition (6). For matrices which do not satisfy this condition, the transition from eq. (7) to eqs. (9) and (10) in Nwoke's paper cannot be made. To see this, we perform a similarity transformation which diagonalizes the matrix $A(x)$. Of course this similarity transformation is dependent on x since the eigenvectors of the matrix $A(x)$ are position-dependent. Multiplying both sides of Nwoke's eq. (7) by $P(x)^{-1}$ from the left and $P(x)$ from the right, we get

$$P(x)^{-1}\exp\left\{\int_{x_0}^{x} A(t)\,dt\right\}P(x) = \alpha + \beta(\lambda(x)). \tag{7}$$

In this equation $P(x)$ is a 2×2 matrix whose columns are the components of eigenvectors of $A(x)$, and $\lambda(x)$ is a diagonal 2×2 matrix whose diagonal elements are the eigenvalues of $A(x)$. In order for the left-hand side of this equation to be equal to that in eqs. (9) and (10) in the paper in question, the matrix $P(x)$ must diagonalize the matrix $A(y)$ for arbitrary values of x and y. This can be seen by expanding the exponential in its power series and trying to reduce each term in the series to a diagonal form. Thus we obtain the following condition on $Z(x)$ and $Y(x)$.

$$Z(x)/Y(x) = \text{constant} = Z(x_0)/Y(x_0). \tag{8}$$

Finally, note that eqs. (14) and (15) do not satisfy eqs. (1) and (2) in Nwoke's paper unless this condition is met. The appendix carries out this verification but this condition was implicitly assumed in getting this result. Also the solutions to the given examples do not satisfy eqs. (1) and (2), as can be seen by direct differentiation. For lines satisfying this condition, the method is an elegant one. The solution to the general case where $Z(x)$ and $Y(x)$ are not related is very complex and involves an infinite ordered series in the matrix $A(x)$. To be explicit, the general solution to eqs. (1) and (2) in Nwoke's paper can be written formally as

$$(x) = X\exp\int_{x_0}^{x} A(t)\,dt \tag{9}$$

where the operator X orders a product $A(x_1)A(x_2)\cdots A(x_n)$, the arguments x_i appearing in ascending order from right to left. For a more detailed discussion standard books on quantum field theory or quantum many-body theory may be consulted, for example [1]. A perturbation expansion may be fruitful for problems where the series (which is known in quantum field theory as the Dyson series) converges.

References

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Comments on "TE and TM Modes of Some Triangular Cross-Section Waveguides Using Superposition of Plane Waves"

Jingjun Zhang and Junmei Fu

In the above paper,[1] Overfelt and White found the exact transverse electric and magnetic mode solution of four triangular cross-section waveguides: 1) equilateral; 2) 30°, 30°, 120°; 3) isosceles right; and 4) 30°, 60° right triangular. But the work of Prof. Lin Weigan some years ago [1] should not be neglected. His results for 30°, 60° right triangular waveguides are as follows.

With the coordinate system in Fig. 1 with a 30° angle at the origin, for TE modes,

$$H_z = \cos\frac{l\pi x}{a}\cos\frac{(m-n)\pi y}{\sqrt{3}a} + \cos\frac{m\pi x}{a}\cos\frac{(n-l)\pi y}{\sqrt{3}a} + \cos\frac{n\pi x}{a}\cos\frac{(l-m)\pi y}{\sqrt{3}a}, \qquad l+m+n=0.$$

The cutoff wavenumbers for the TE modes are

$$k_c = \frac{2\pi}{\sqrt{3}a}\sqrt{m^2+mn+n^2}$$

Manuscript received August 21, 1990.
The authors are with the Department of Information and Control Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi, People's Republic of China.
IEEE Log Number 9041938.

[1] P. L. Overfelt and D. J. White, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 161–167, Jan. 1986.

0018-9480/91/0300-0612$01.00 ©1991 IEEE